8.1 (Local) Stratification

Test for the following Datalog programs if the stratification condition is violated. Are the programs still loosely/locally stratifiable? Discuss your answer with the recursion graph.

a)  
\[
\begin{align*}
p(X) & :- \text{NOT } q(X), \; t(X). \\
q(X) & :- r(X,Y), \text{NOT } s(X). \\
r(X,Y) & :- r(Y,X), \; t(Y). \\
s(X) & :- t(X), \text{NOT } r(X). \\
s(a) & \; \text{r(a, b).} \\
t(a) & \; \text{t(b).}
\end{align*}
\]

b)  
\[
\begin{align*}
p(X) & :- q(X,Y), \; t(Y). \\
q(X,Y) & :- s(X,Y). \\
s(X,b) & :- \text{NOT } p(X). \\
s(a,b) & \; \text{t(a).} \\
q(a, a) & \; \text{q(a, a).}
\end{align*}
\]

8.2 Herbrand Models

Given the following DATALOG program:

\[
\begin{align*}
t(X) & :- p(X). \\
t(X) & :- q(Y,X). \\
q(X,Y) & :- p(X), \; q(Y,X). \\
q(X,X) & :- t(X). \\
q(c,a) & \; \text{p(a).} \\
p(a) & \; \text{p(b).}
\end{align*}
\]

a) Find a corresponding Herbrand base.
b) Calculate the minimal Herbrand model.
c) Find two more not minimal Herbrand models.
8.3 Stratification

Given the following Datalog rules that characterize a train network, defined by the relation link:

- **R1**: cutpoint(X,A,B):- connected(A,B),station(X),NOT circumvent(X,A,B).
- **R2**: circumvent(X,A,B):- station(X),linked(A,B),X≠A,X≠B.
- **R3**: circumvent(X,A,B):- circumvent(X,A,C), circumvent(X,C,B).
- **R4**: linked(A,B):- link(A,B).
- **R5**: linked(A,B):- link(B,A).
- **R6**: connected(A,B):- linked(B,A).
- **R7**: connected(A,B):- connected(A,C), linked(C,B).
- **R8**: exists_cutpoint(A,B):- station(X), cutpoint(X,A,B).
- **R9**: safely_connected(A,B):- connected(A,B),NOT exists_cutpoint(A,B).
- **R10**: station(X):- linked(X,Y).

The relations can be interpreted as follows:

- **cutpoint(X,A,B)**: Each way from A to B passes X
- **connected(A,B)**: A and B are directly or indirectly connected
- **circumvent(X,A,B)**: There is a connection between A and B that doesn’t pass X
- **exists_cutpoint(A,B)**: There is a cutpoint for A and B
- **safely_connected(A,B)**: Even if an intermediate station fails (e.g. due to an accident) there is still a connection between A and B.

Extensional relation **link** is given by:

- link(Eschweiler, Düren)
- link(Eschweiler, Aachen)
- link(Aachen, Neuss)
- link(Neuss, Düsseldorf)
- link(Düren, Heimbach)
- link(Düren, Bedburg)
- link(Bedburg, Neuss)

Divide the predicates into strata (layers) and compute the perfect model (i.e. the “natural” minimal Herbrand model) based on the extensional relation link.