7.1 Datalog

a) Given the following database containing fact sets for following predicates.

- Male(X): Person X is male
- Married(X,Y): Person X and Y are married
- Child(X,Y): Person X is a child of person Y
- Lives(X): Person X has not died so far

Please formulate Datalog rules for the following connections:

- Parent
- Sibling
- Aunt
- Daughter
- Brother
- Sister
- Niece
- Only child
- Brother-in-law
- Widow
- Grandchild
- Male Ancestor
- Orphan

b) Given your rule for Parent(X,Y) from a), the rule for same generation (sg(X,Y)) and the following facts:

\[
\text{Parent}(X,Y) : \text{ Solution of a).}
\]
\[
\text{sg}(X,Y) : \text{- Parent}(Z,X), \text{Parent}(Z,Y), X \neq Y.
\]
\[
\text{sg}(X,Y) : \text{- Parent}(X,W), \text{Parent}(Y,Z), \text{sg}(W,Z), X \neq Y.
\]
\[
\text{Child}(\text{Hera}, \text{Kronos}).
\]
\[
\text{Child}(\text{Hera}, \text{Rhea}).
\]
\[
\text{Child}(\text{Kronos}, \text{Gaea}).
\]
\[
\text{Child}(\text{Rhea}, \text{Uranos}).
\]
\[
\text{Child}(\text{Zeus}, \text{Kronos}).
\]

Compute the extension of \( \text{sg}(X,Y) \), write down all intermediate steps leading to new results.

7.2 Datalog, Herbrand Models

Given the following Datalog program

```datalog
%% Rule system
con(X,U) :- edge(X,U).
con(X,U) :- edge(X,Z), con(Z,U).
dcon(X,Y) :- con(X,U), con(Y,U).
```
Compute the minimal Herbrand Model ($F^*$) for the Datalog program, write down all intermediate steps leading to new results.

### 7.3 Datalog & Fixpoints

Consider a tennis tournament database with information about players with predicates

- `beats(X1,X2)`: $X1$ beats $X2$
- `superior(X1,X2)`: $X1$ is superior to $X2$.

Assume that if a player beats another player he is superior to that player and assume that if player 1 beats player 2 and player 2 is superior to player 3 then player 1 is superior to player 3.

a) Construct a set of recursive rules using the above predicates
b) Populate data for beats relation with 10 players playing 3 matches each.
c) Show the computation of the minimal Herbrand Model (computing the least fixpoint of the consequence operator $T_D$) using this data.
d) Does the computation of the superior table have a fixpoint? Why or why not? Is this always the case? Explain!